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INCREMENTS OF THE ARGUMENT AND FUNCTION. THE CONCEPT OF THE DERIVATIVE. RULES FOR CALCULATING DERIVATIVES

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Abstract

This article discusses the increment of an argument and a function, the concept of the derivative, and the rules for calculating derivatives, which are among the fundamental concepts of mathematical analysis. The importance of the increments of the argument and the function in describing the process of function variation is highlighted. In addition, the basic rules for derivative calculation are explained, including the methods for finding the derivatives of sums, differences, products, quotients, and composite functions. This topic serves as an important theoretical foundation for the in-depth study of mathematical analysis and for solving practical problems.

Keywords

argument increment, function increment, derivative, limit, mathematical analysis, differential calculus, derivative calculation rules, composite function, differentiable function.

ПРИРАЩЕНИЕ АРГУМЕНТА И ФУНКЦИИ. ПОНЯТИЕ ПРОИЗВОДНОЙ. ПРАВИЛА ВЫЧИСЛЕНИЯ ПРОИЗВОДНОЙ

Аннотация

В данной статье рассмотрены приращение аргумента и функции, понятие производной, а также правила вычисления производной, являющиеся одними из основных понятий математического анализа. Показана роль приращения аргумента и функции при описании процесса изменения функции. Кроме того, разъяснены основные правила вычисления производной, включая правила нахождения производной суммы, разности, произведения, частного и сложной функции. Данная тема служит важной

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теоретической основой для углублённого изучения математического анализа и решения практических задач.

Ключевые слова

приращение аргумента, приращение функции, производная, предел, математический анализ, дифференциальное исчисление, правила вычисления производной, сложная функция, дифференцируемая функция.

ARGUMENT VA FUNKTSIYA ORTTIRMASI. HOSILA TUSHUNCHASI. HOSILANI HISOBLASH QOIDALARI

Annotatsiya

Ushbu maqolada matematik analizning asosiy tushunchalaridan biri boʻlgan argument va funksiya orttirmasi, hosila tushunchasi hamda hosilani hisoblash qoidalari yoritilgan. Funksiyaning oʻzgarish jarayonini tavsiflashda argument va funksiya orttirmalarining ahamiyati koʻrsatilgan.Shuningdek, hosilani hisoblashning asosiy qoidalari, jumladan, yigʻindi, ayirma, koʻpaytma, boʻlinma va murakkab funksiyaning hosilasini topish usullari tushuntirilgan. Mazkur mavzu matematik analizni chuqur oʻrganish hamda amaliy masalalarni yechishda muhim nazariy asos boʻlib xizmat qiladi.

Kalit soʻzlar

argument orttirmasi, funksiya orttirmasi, hosila, limit, matematik analiz, differensial hisob, hosilani hisoblash qoidalari, murakkab funksiya, differensiallanuvchi funksiya.

Introduction. Argument va funktsiya orttirmasi. Faraz qilaylik, sonlar oʻqida biror x_1 nuqta berilgan boʻlsin. Bu nuqta yangi x_2 nuqtaga koʻchirilsin. U holda, x_1 va x_2 nuqtalar orasida masofali farq hosil boʻladi. Ana shu masofali farqning uzunligi $x_2 - x_1$ dan iborat boʻladi. Bu farqni

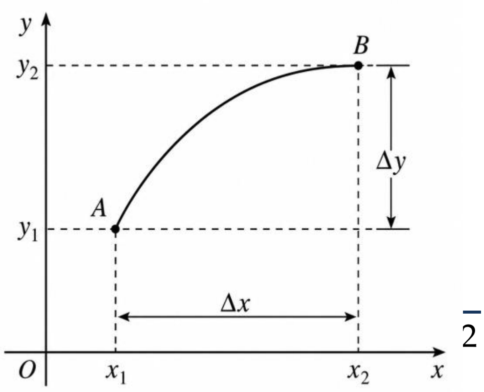
Δx bilan belgilasak,

$x_2 - x_1 = \Delta x$ (1)

boʻladi. Δx ni ***orttirma*** deb ataymiz.

Dekart koordinatalari sistemasida $y = f(x)$ funktsiyaning grafigi berilgan boʻlsin.

Funktsiyaning ixtiyoriy A nuqtasining absissa x_1 dan va ordinatasi y_1 dan iborat. Agar A nu grafik boʻylab B nuqtagacha siljisa, B nuqtanin



absissasi x_2 ,ordinatasi esa y_2 dan iborat bo’ladi.

U holda, absissadagi farq $x_2 - x_1 = \Delta x$,
ordinatadagi farq esa $y_2 - y_1 = \Delta y$ dan
iborat boʻladi. Bundan quyidagicha mulohaza yuritish mumkin: funktsiyaning argumenti Δx orttirma olsa, funktsiya ham Δy orttirma oladi. Demak, $y= f(x)$ funktsiyaning orttirmasi argumentning Δx orttirmasi bilan aniqlanadi.

- Mavzuga oid adabiyotlarning tahlili (Literature review).

Ta’rif: $y=f(x)$ funktsiya uchun uning aniqlanish sohasidan olingan x_1 va x_2 argumentlari qiymatlarining ayirmasi ***argumentning orttirmasi*** deyiladi.

Ta’rif: $y=f(x)$ funktsiyaning qiymatlar sohasi (oʻzgarish sohasi)dan olingan x_1 va x_2 argumentlariga mos $y_1 = f_1(x)$va $y_2 = f_2(x)$ orasidagi ayirma ***funktsiyaning orttirmasi*** deyiladi.

Agar x argument Δx orttirma olgan boʻlsa, argumentning ortgan qiymati $x + \Delta x$, funktsiyaning unga mos qiymati esa quyidagidan iborat boʻladi:

$y + \Delta y = f(x + \Delta x)$ funktsiyaning orttirmasini topish uchun uning ortgan qiymatidan dastlabki qiymatini ayirish lozim, yaʻni

$$\begin{array}{r} y + \Delta y = f(x + \Delta x) \\ - \; y = f(x) \\ \hline \Delta y = f(x + \Delta x) - f(x) \end{array}$$

Hosila tushunchasi.(Hos[illegible]q ta’rif – Hosila.Hosila tushunchasining ta’rifi maqolamizda to‘liq keltirilgan)

Ta’rif: $y=f(x)$ funktsiyaning x nuqtadagi ***hosilasi*** deb, funktsiyaning shu nuqtadagi orttirmasi $\Delta f(x)$ *yoki* Δy ning argument orttirmasi Δx ga nisbatining $\Delta x \to 0$ ga intilgandagi limitiga aytiladi.

Demak, ta’rifga asosan

$$y' = y'_x = f' = f'(x) = \frac{dy}{dx} = \frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

- Tadqiqot metodologiyasi (Research Methodology).

Berilgan $y=f(x)$ funktsiyadan hosila olish amali shu funktsiyani ***differentsiallash*** deyiladi.

Birorta oraliqning har bir nuqtasida hosilaga ega bo‘lgan funktsiya shu oraliqda ***differentsiallanuvchi*** deyiladi.

$y=f(x)$ funktsiyaning hosilasi uchun quyidagi belgilashlar ishlatiladi:

$$y', y'_x, f', f'(x), \frac{dy}{dx}, \frac{df(x)}{dx}$$

$y=f(x)$ funktsiyaning hosilasini hisoblash differesiallashning quyidagi bosqichlari bo‘yicha bajariladi:

Differentsiallashning to‘rt bosqichi

1^0. x argumentga Δx orttirma berib va funktsiya ifodasini x o‘rniga orttirilgan qiymat $x+\Delta x$ ni qo‘yib, funktsiyaning orttirilgan qiymati topiladi:

$$y+\Delta y = f(x+\Delta x)$$

2^0. Funktsiyaning orttirilgan qiymatidan uning boshlang‘ich qiymatini ayirib, funktsiya orttirmasi topiladi:

$$\Delta y = f(x+\Delta x) - f(x)$$

3^0. Funktsiyaning orttirmasi Δy ni argument orttirmasi Δx ga bo‘lib, quyidagi nisbat tuziladi:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

4^0. Bu nisbatning $\Delta x \to 0$ dagi limiti topiladi:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Hosilani hisoblash qoidalari. Differensiallash qoidalari.Differensiallash – hosilani topish amalidir.

1. $\boldsymbol{y = x}$ O‘zgaruvchining shu o‘zgaruvchiga ko‘ra hosilasi 1 ga teng.

Isbot: Bu teorema yuqorida keltirilgan – differentsiallashning to‘rt bosqichi asosida isbotlanadi, bu bosqichlarni ketma- ket qo‘llaymiz.

$$\begin{array}{r} y + \Delta y = x + \Delta x \\ - \quad y = x \\ \hline \Delta y = \Delta x \end{array}$$

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$$\frac{\Delta y}{\Delta x} = \frac{\Delta x}{\Delta x} \quad \rightarrow \quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} \quad \rightarrow \quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 1$$

$$\rightarrow \qquad y' = 1 \quad \rightarrow \quad y' = (x)' = 1$$

1 – formulani ushbu $y' = (uv)' = u'v + v'u$, $(x^n)' = \mathrm{n} \cdot x^{n-1}$ formulalarni bilgan holda xususiy holda isbotlaymiz.

$$y = x \quad \rightarrow \quad y = 1 \cdot x \quad \rightarrow \quad y' = (1 \cdot x)' = (1)^1 \cdot x + (x)' \cdot 1 \quad \rightarrow$$

$$(x^n)' = \mathrm{n} \cdot x^{n-1} \;\; ga\ ko`ra \;\; (x^1)' = 1 \cdot x^{1-1} = 1 \cdot 1^0 \quad \rightarrow$$

$$a^0 = 1 \;\; dan \;\; 1^0 = 1 \;\; bo`ladi, demak \quad \rightarrow$$

$y' = (x)' = 1$ isbotlandi.

2. O‘zgarmas sonning hosilasi 0 ga teng, ya’ni $y' = f'(x) = c' = 0$

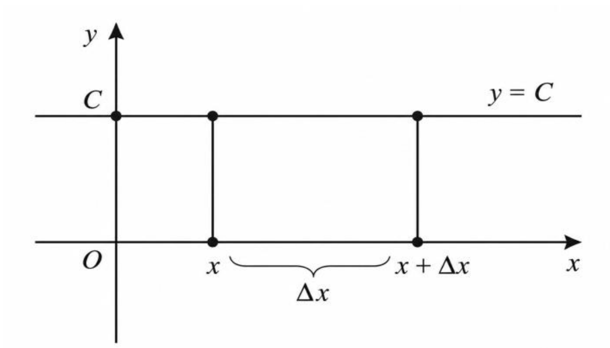
Isbot: Bu teorema yuqorida keltirilgan – differentsiallashning to‘rt bosqichi asosida isbotlanadi, bu bosqichlarni ketma- ket qo‘llaymiz.

Bizga , $y = f(x) = c$ $(c - o'zgarmas\ son)$ funktsiya berilgan bo‘lsin.

$$y + \Delta y = c \quad \rightarrow \quad \Delta y = c - y = c - c = 0 \quad \rightarrow \quad \frac{\Delta y}{\Delta x} = \frac{0}{\Delta x} = 0 \quad \rightarrow \quad y'$$

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 0 = 0 \quad \rightarrow \quad y' = f'(x) = c' = 0$$

2 – formulani hosila ta’rifidan foydalanib yana bir isbotlaylik



ar qanday x qiymat uchun funksiyaning x dan x+Δx ga o‘tganda ham f(x)=C va f(x+Δx)=C bo`ladi.

Hosila ta’rifiga ko‘ra: $y' = c' = \frac{f(x+\Delta x) - f(x)}{\Delta x}$ $\rightarrow$ Doimiy funksiya uchun:

$$\rightarrow \quad \frac{c - c}{\Delta x} = \frac{0}{\Delta x} = 0$$

Demak, doimiy funksiyaning hosilasi har qanday nuqtada nolga teng: (C)' = 0.

3. O‘zgarmas c soni bilan biror funktsiya ko‘paytmasining hosilasi shu son bilan funktsiya hosilasining ko‘paytmasiga teng, ya’ni:

$y = c \cdot u \quad \rightarrow y' = (c \cdot u)' = c \cdot u'$

Isbot: Bu teorema yuqorida keltirilgan – differentsiallashning to‘rt bosqichi asosida isbotlanadi, bu bosqichlarni ketma- ket qo‘llaymiz.

Faraz qilaylik, $u = f(x)$ bo‘lsin. U holda $y = cu = cf(x)$ bo‘ladi.

1^0. $y + \Delta y = cf(x + \Delta x)$

2^0. $\Delta y = cf(x + \Delta x) - cf(x) = c[f(x + \Delta x) - f(x)] = c \cdot \Delta u$

3^0. $\frac{\Delta y}{\Delta x} = \frac{c \cdot \Delta u}{\Delta x}$

4^0. $y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left(c \cdot \frac{\Delta u}{\Delta x}\right) = c \cdot \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = c \cdot u'$

Demak, $y = c \cdot u \quad \rightarrow y' = (c \cdot u)' = c \cdot u'$

3 – formulani, quyidagi $y' = (uv)' = u'v + v'u$ formulani bilgan holda yangicha isbotlaymiz.

$y = c \cdot u \quad (c = const\,, c \neq 0) \quad \rightarrow y' = (c \cdot u)' = c \cdot u'$

Isbot: $(c \cdot u)' = c'u + u'c = 0 \cdot u + c \cdot u' = c \cdot u'$

4. Agar $u = u(x) va\ v = v(x)$ bo‘lsa , ikki funktsiya yig’indisi(ayirmasi) hosilasi shu funktsiyalar hosilalarining yig’indisi(ayirmasi)ga teng.

Isbot: Bu teorema yuqorida keltirilgan – differentsiallashning to‘rt bosqichi asosida isbotlanadi, bu bosqichlarni ketma- ket qo‘llaymiz.

$$y = u + v \quad \rightarrow \quad y + \Delta y = u + \Delta u + v + \Delta v \quad \rightarrow \quad shu\ tenglikdan\ \ y = u + v \quad ni\ ayirsak \quad \rightarrow \quad \Delta y = \Delta u + \Delta v \quad \rightarrow \quad \frac{\Delta y}{\Delta x} = \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta x} \quad \rightarrow \quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} + \lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} \quad \rightarrow \quad y' = u' + v'$$

$y = u - v \quad \rightarrow$ (1) $\quad y + \Delta y = u + \Delta u - (v + \Delta v) \quad \rightarrow$ (2) $y = u - v$ (1) - (2) ni ayiramiz. $y + \Delta y - y = u + \Delta u - (v + \Delta v) - (u - v) \quad \rightarrow$

$y + \Delta y - y = u + \Delta u - v - \Delta v - u + v \quad \rightarrow \quad o'xshash\ hadlarni\ qisqartirib$

$$\Delta y = \Delta u - \Delta v \quad \rightarrow \frac{\Delta y}{\Delta x} = \frac{\Delta u}{\Delta x} - \frac{\Delta v}{\Delta x} \quad \rightarrow \quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} - \lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} \quad \rightarrow \quad y' = u' - v'$$

4 – formulani hosila ta’rifidan foydalanib yana bir isbotlaymiz.

Agar $u = u(x) va\ v = v(x)$ bo`lsa, $(u(x) + v(x))' = u'(x) + v'(x) \quad \rightarrow$

bizga ma’lum $y = f(x)$ $ning\ hosilasi$ $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ $ga\ teng$

$$(u(x) + v(x))' = \lim_{\Delta x \to 0} \frac{u(x + \Delta x) + v(x + \Delta x) - u(x) - v(x)}{\Delta x} \quad \rightarrow$$

$$\lim_{\Delta x \to 0} \frac{u(x + \Delta x) - u(x) + v(x + \Delta x) - v(x)}{\Delta x} \quad \rightarrow$$

$$\lim_{\Delta x\to 0}\frac{u(x+\Delta x)-u(x)}{\Delta x}+\lim_{\Delta x\to 0}\frac{v(x+\Delta x)-v(x)}{\Delta x}\rightarrow u'(x)+v'(x)$$

$(u(x)-v(x))'=u'(x)-v'(x)$ ni oʻzingizga topshiraman.

5. Ikki funktsiya koʻpaytmasining hosilasi birinchi funktsiya hosilasini ikkinchi funktsiyaga koʻpaytmasiga qoʻshish ikkinchi funktsiya hosilasini birinchi funktsiyaga koʻpaytmasiga teng. $y=u\cdot v$ u va v $x-ning\ funktsiyalari$

$x\ ga\ \Delta x$ orttirma beramiz, u holda $u,v\ va\ y\ funktsiyalar$ mos holda $\Delta u,\Delta v\ va\ \Delta y$ orttirmalarni oladi.

Isbot: Bu teorema yuqorida keltirilgan – differentsiallashning toʻrt bosqichi asosida isbotlanadi, bu bosqichlarni ketma- ket qoʻllaymiz.

$$y=u\cdot v\ \rightarrow\ y+\Delta y=(u+\Delta u)(v+\Delta v)\ \rightarrow\ y+\Delta y=uv+u\Delta v+v\Delta u+\Delta u\Delta v$$

$$\rightarrow\ shu\ tenglikdan\ \ y=u\cdot v\ \ ni\ ayirsak\ \rightarrow\ \Delta y$$

$$=u\Delta v+v\Delta u+\Delta u\Delta v\ \rightarrow\ \frac{\Delta y}{\Delta x}=\frac{u\Delta v}{\Delta x}+\frac{v\Delta u}{\Delta x}+\frac{\Delta u\Delta v}{\Delta x}\ \rightarrow$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{u\Delta v}{\Delta x}+\lim_{\Delta x\to 0}\frac{v\Delta u}{\Delta x}+\lim_{\Delta x\to 0}\frac{\Delta u\Delta v}{\Delta x}\ \rightarrow$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=u\cdot\lim_{\Delta x\to 0}\frac{\Delta v}{\Delta x}+v\cdot\lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x}+\lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x}\cdot\lim_{\Delta x\to 0}\Delta v$$

$u\ va\ v$ funktsiyalar Δx ga bogʼliq emas, shuning uchun ular limit ishorasidan tashqariga chiqarilgan:

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=y',\quad \lim_{\Delta x\to 0}\frac{\Delta v}{\Delta x}=v',\quad \lim_{\Delta x\to 0}\frac{\Delta u}{\Delta x}=u',\quad \lim_{\Delta x\to 0}\Delta v=0$$

$$y=uv\ \rightarrow\ y'=u'v+v'u\ \rightarrow\ (uv)'=u'v+v'u$$

Tahlil va natijalar (Analysis and results).

5 – formulani hosila taʼrifidan foydalanib yana bir isbotlaymiz.

Agar $u=u(x)va\ v=v(x)$ boʻlsa,

$(u(x)\cdot v(x))'=$

$\lim_{\Delta x\to 0}\frac{u(x+\Delta x)v(x+\Delta x)-u(x)v(x)}{\Delta x}\ \rightarrow\ u(x)v(x+\Delta x)\ ni\ \ bir\ qo'shib,bir\ ayiraman$

$$\lim_{\Delta x\to 0}\left(\frac{u(x+\Delta x)v(x+\Delta x)-u(x)v(x+\Delta x)}{\Delta x}+\frac{u(x)v(x+\Delta x)-u(x)v(x)}{\Delta x}\right)\rightarrow$$

$$\lim_{\Delta x\to 0}\left(\frac{v(x+\Delta x)(u(x+\Delta x)-u(x))}{\Delta x}+\frac{u(x)(v(x+\Delta x)-v(x))}{\Delta x}\right)\rightarrow$$

$$\lim_{\Delta x\to 0}\left(\frac{v(x+\Delta x)(u(x+\Delta x)-u(x))}{\Delta x}+\lim_{\Delta x\to 0}\frac{u(x)(v(x+\Delta x)-v(x))}{\Delta x}\right)\rightarrow$$

$\Delta x\rightarrow 0$ ga intilganda $v(x+\Delta x)\rightarrow v(x)$ ga intiladi, $\lim_{\Delta x\to 0}\frac{u(x+\Delta x)-u(x)}{\Delta x}=u'(x)$ ga teng boʻladi.

$\Delta x\rightarrow 0$ ga intilganda $u(x)\rightarrow u(x)$ ga teng,chunki Δx ga bogʻliq emas.

$\lim_{\Delta x \to 0} \frac{v(x+\Delta x)-v(x)}{\Delta x} = v'(x)$ ga teng bo'ladi.Demak $v(x)u'(x) + u(x)v'(x)$ isbotlandi.

5 - formulani ushbu $y = \ln u \to y' = (\ln u)' = \frac{u'}{u}$ formulani bilgan holda logarifmik differensiallash usuli bilan yangicha isbotlaymiz.

$y' = (uv)' = u'v + v'u$ isboti

1. $y = u \cdot v$
2. $lny = \ln(u \cdot v)$ funktsiyani ikkala tomonini natural logarifmlaymiz
3. $(lny)' = (\ln(u \cdot v))'$ hosila olamiz
4. $(lny)' = (lnu + lnv)'$ logarifm xossasidan foydalandik
5. $(lny)' = (lnu)' + (lnv)'$
6. $\frac{y'}{y} = \frac{u'}{u} + \frac{v'}{v}$ ushbu $y = \ln u \to y' = (\ln u)' = \frac{u'}{u}$ formulani ishlatdik
7. $\frac{y'}{y} = \frac{u'v+v'u}{uv}$ umumiy maxraj topamiz
8. $y'(uv) = (u'v + v'u)y$ proporsiya
9. $y' = y \cdot \frac{u'v+v'u}{uv}$ y' ni topamiz
10. $y' = uv \cdot \frac{u'v+v'u}{uv}$ berilgan y ni o'rniga $y = u \cdot v$ qiymatni qo'yamiz
11. $y' = (uv)' = u'v + v'u$ qisqartirishdan so'ng,formula hosil bo'ladi.

6. Ikki funktsiya bo'linmasining hosilasi shunday kasrga teng,uning surati bo'luvchi bilan bo'linuvchi hosilasining ko'paytmasiga ayiruv bo'linuvchi bilan bo'luvchi hosilasining ko'paytmasiga teng,maxraji esa bo'luvchining kvadratidan iborat.

$y = \frac{u}{v}$ funktsiya berilgan bo'lsin. Bu yerda $v \neq 0$

Isbot: Bu teorema yuqorida keltirilgan – differentsiallashning to'rt bosqichi asosida isbotlanadi, bu bosqichlarni ketma- ket qo'llaymiz.

$$y + \Delta y = \frac{u + \Delta u}{v + \Delta v} \to$$

$$\Delta y = \frac{u + \Delta u}{v + \Delta v} - \frac{u}{v} = \frac{v(u + \Delta u) - u(v + \Delta v)}{v(v + \Delta v)} = \frac{uv + v\Delta u - uv - u\Delta v}{v(v + \Delta v)} \to$$

$$\Delta y = \frac{v\Delta u - u\Delta v}{v(v + \Delta v)} \quad \to \quad \frac{\Delta y}{\Delta x} = \frac{\frac{\Delta u}{\Delta x} \cdot v - \frac{\Delta v}{\Delta x} \cdot u}{v(v + \frac{\Delta v}{\Delta x} \cdot \Delta x)} \to$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{x\to 0}\frac{\frac{\Delta u}{\Delta x}\cdot v-\frac{\Delta v}{\Delta x}\cdot u}{v(v+\frac{\Delta v}{\Delta x}\cdot\Delta x)}=\frac{v\cdot\lim\limits_{\Delta x\to 0}\frac{\Delta u}{\Delta x}-u\cdot\lim\limits_{\Delta x\to 0}\frac{\Delta v}{\Delta x}}{v(v+\lim\limits_{\Delta x\to 0}(\frac{\Delta v}{\Delta x}\cdot\Delta x))}\to$$

$$y'=\frac{u'v-v'u}{v(v+v'\cdot 0)}=\frac{u'v-v'u}{v^2}$$

$$y'=\left(\frac{u}{v}\right)'=\frac{u'v-v'u}{v^2}$$

Xulosa va takliflar (Conclusion/Recommendations).

6 – formulani hosila ta'rifidan foydalanib yana bir isbotlaymiz.

Agar $u=u(x) va\ v=v(x)$ bo`lsa, $\left(\frac{u(x)}{v(x)}\right)'\to hosila\ ta'rifidan\to$

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\quad ga\ teng, demak\to$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\frac{u(x+\Delta x)}{v(x+\Delta x)}-\frac{u(x)}{v(x)}}{\Delta x}\to\frac{u(x)}{v(x+\Delta x)}\ ni\ bir\ qo'shib, bir\ ayiramiz\to$$

$$\lim_{\Delta x\to 0}\frac{\frac{u(x+\Delta x)}{v(x+\Delta x)}-\frac{u(x)}{v(x+\Delta x)}+\frac{u(x)}{v(x+\Delta x)}-\frac{u(x)}{v(x)}}{\Delta x}\to$$

$$\lim_{\Delta x\to 0}\frac{\frac{u(x+\Delta x)-u(x)}{v(x+\Delta x)}+u(x)\left(\frac{1}{v(x+\Delta x)}-\frac{1}{v(x)}\right)}{\Delta x}\to$$

$$\lim_{\Delta x\to 0}\frac{\frac{u(x+\Delta x)-u(x)}{v(x+\Delta x)}-u(x)\left(\frac{v(x+\Delta x)-v(x)}{v(x)\cdot v(x+\Delta x)}\right)}{\Delta x}\to$$

$$\lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x \cdot v(x+\Delta x)} - \lim_{\Delta x \to 0} \frac{u(x)(v(x+\Delta x) - v(x))}{\Delta x \cdot v(x+\Delta x) \cdot v(x)} \rightarrow$$

Birinchi ifoda $\Delta x \rightarrow 0$ $ga\ intilganda$ $v(x+\Delta x) \rightarrow v(x) ga\ intiladi$ $\rightarrow$

$$\lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} = u'(x)\ ga\ teng\ hosila\ ta'rifidan$$

Ikkinchi ifoda $\Delta x \rightarrow 0$ $ga\ intilganda$ $v(x+\Delta x) \rightarrow v(x) ga\ intiladi$ $\rightarrow$

$$\lim_{\Delta x \to 0} \frac{v(x+\Delta x) - v(x)}{\Delta x} = v'(x)\ ga\ teng\ hosila\ ta'rifidan$$

Demak quyidagicha yozib olamiz. $\frac{u'(x)}{v(x)} - \frac{u(x) \cdot v'(x)}{v^2(x)} = \frac{v(x)u'(x) - u(x)v'(x)}{v^2(x)}$ $\rightarrow$

$$y = \frac{u(x)}{v(x)} \rightarrow \left(\frac{u(x)}{v(x)}\right)' = \frac{v(x)u'(x) - u(x)v'(x)}{v^2(x)} \rightarrow$$

Isbotlandi.

6 - formulani ushbu $y = \ln u \rightarrow y' = (\ln u)' = \frac{u'}{u}$ formulani bilgan holda logarifmik differensiallash usuli bilan yangicha isbotlaymiz. $y' = \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$ isboti

1. $y = \frac{u}{v}$
2. $lny = ln\left(\frac{u}{v}\right)$ funktsiyani ikkala tomonini natural logarifmlaymiz
3. $(lny)' = \left(ln\left(\frac{u}{v}\right)\right)'$ ikkala tomonidan hosila olamiz
4. $(lny)' = (lnu - lnv)'$ logarifm xossasidan foydalandik
5. $(lny)' = (lnu)' - (lnv)'$ ayirmaning hosilasi
6. $\frac{y'}{y} = \frac{u'}{u} - \frac{v'}{v}$ ushbu $y = \ln u \rightarrow y' = (\ln u)' = \frac{u'}{u}$ formulani ishlatdik
7. $\frac{y'}{y} = \frac{u'v - v'u}{uv}$ umumiy maxraj topamiz
8. $y'(uv) = (u'v - v'u)y$ proporsiya
9. $y' = y \cdot \frac{(u'v - v'u)}{uv}$ y' ni topamiz
10. $y' = \frac{u}{v} \cdot \frac{(u'v - v'u)}{uv}$ berilgan y ni oʻrniga $y = \frac{u}{v}$ qiymatni qoʻyamiz
11. $y' = \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$ qisqartirishdan soʻng, formula hosil boʻladi.

FOYDALANILGAN ADABIYOTLAR:

1. Oliy matematika (Hosila va differentsial) Uslubiy qo'llanma. Sultonov J.S. Samarqand -2010
2. Matematika(oliy ta'lim muassasalariga kiruvchilar uchun qo'llanma).Dusmuratov M.B.Toshkent-2018
3. M.Usmanov.Matematika(oliy o'quv yurtlariga kiruvchilar uchun)ma'lumotnoma.1-qism. "NAVRO'Z" Toshkent 2017.
4. https://youtube.com/@jdpuuz.Jizzax davlat pedagogika universiteti.Hosila olish qoidalari, video dars – 2024.Ismatov N.
5. https://youtube.com/@SherzodNabiyev_Matematika