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IRRATSIONAL FUNKSIYALAR BILAN BOG'LIQ EKSTREMAL MASALALAR VA TENGLAMALARNI YECHISHNING GEOMETRIK USULLARI

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Annotatsiya

Ushbu maqolada irratsional tenglamalarni geometrik usulda oson va qulay yechish yo'llari keltirilgan.

Kalit so'z

Tenglama,usul,geometrik,yechim.

Аннотация

В данной статье представлены простые и удобные способы решения иррациональных уравнений с использованием геометрических методов.

Ключевые слова

уравнение, метод, геометрический, решение.

Abstract

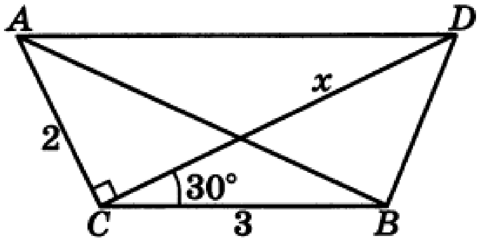
This article presents easy and convenient ways to solve irrational equations using geometric methods.

Keywords

Equation, method, geometric, solution.

1-misol. $f(x)=\sqrt{x^4+4}+\sqrt{x^2-3x\sqrt{3}+9}$ funksiyaning eng kichik qiymatini toping.

Yechish. 1–chizmada $\Delta ACD\ (AC=2, CD=x,\ \angle ACD=90^0)$ va $\Delta BCD\ (BC=3, CD=x,\ \angle BCD=30^0)$



1-chizma

ΔACD dan Pifagor teoremasiga koʻra $AD=\sqrt{x^2+4}$

ΔBCD dan kosinuslar teoremasiga koʻra $DB=\sqrt{x^2+9-3x\sqrt{3}}$

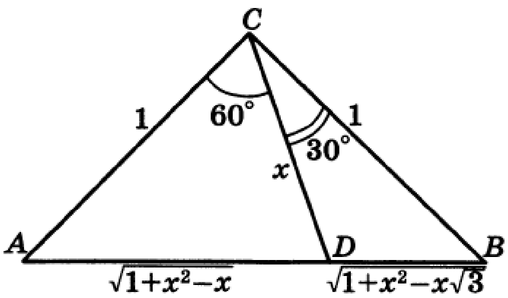
$\min f(x)=\min(AD+DB)=AB$

ABC uchburchakdan kosinuslar teoremasiga koʻra

$AB=\sqrt{2^2+3^2-2\cdot 2\cdot 3\cos 120^0}=\sqrt{19}$.

2-misol. x ning qanday qiymatlarida $f(x)=\sqrt{1+x^2-x}+\sqrt{1+x^2-x\sqrt{3}}$ funksiya oʻzining eng kichik qiymatiga erishadi?

Yechish. 2 –chizmadan ABC, ACD va BCD uchburchaklar yuzini topamiz.



2-chizma

$S_{\Delta ABC}=\frac{1}{2}$.

$S_{\Delta ACD}=\frac{1}{2}\cdot 1\cdot x\cdot \sin 60^0=\frac{x\sqrt{3}}{4}$.

$S_{\Delta BCD}=\frac{1}{2}\cdot 1\cdot x\cdot \sin 30^0=\frac{x}{4}$.

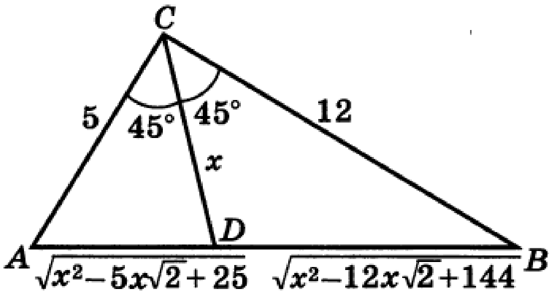
Demak, $\frac{1}{2}=\frac{x\sqrt{3}}{4}+\frac{x}{4}$; $x=\sqrt{3}-1$.

Javob: $\sqrt{3}-1$.

3-misol. $\sqrt{x^2-5x\sqrt{2}+25}+\sqrt{x^2-12x\sqrt{2}+144}=13$ tenglamani yeching.

Yechish. Bu tenglamaning anʼanaviy usulda yechish uchun har tomoni ildizlar yoʻqolmaguncha kvadratga koʻtariladi. Tenglamani geometrik usulda yechish qisqa muddatda amalga oshiriladi. 3–chizmada ΔABC dan Pifagor teoremasiga koʻra $AB=13$.

$D\in AB$, chunki $\min f(x)=\min(AD+DB)=AB$.



3-chizma

Bunda $f(x)=\sqrt{x^2-5x\sqrt{2}+25}+\sqrt{x^2-12x\sqrt{2}+144}$ x ning qiymatini topish uchun 2-misoldagidek ish yuritamiz. Agar tenglama ildizlarga ega bo‘lsa, ular musbat bo‘lishni aniqlaymiz. ($x\le 0$ da tenglama chap qismining qiymati 12 dan kichik emas).

$$S_{\Delta ABC}=\frac{1}{2}\cdot 5\cdot 12=30 .$$

$$S_{\Delta ACD}=\frac{1}{2}\cdot 5\cdot x\cdot \sin 45^0=\frac{5x\sqrt{2}}{4} .$$

$$S_{\Delta BCD}=\frac{1}{2}\cdot 12\cdot x\cdot \sin 45^0=3x\sqrt{2} .$$

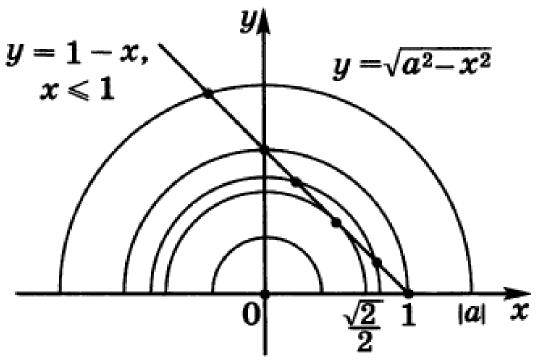
Demak, $30=\frac{5x\sqrt{2}}{4}+3x\sqrt{2}$; $x=\frac{60\sqrt{2}}{17}$.

Javob: $\frac{60\sqrt{2}}{17}$.

4-misol. a parametrning har bir qiymatida $\sqrt{a^2-tg^2z}=1-tgz$ tenglamada tgz nechta har xil qiymatlar qabul qiladi.

Yechish. Bu oddiy tenglama emas. Uni an’anaviy usulda yechish oson emas, agar tgz ni x bilan almashtirsak, uni geometrik usulda yechish oson bo‘ladi.

4-chizmada $y=\sqrt{a^2-x^2}$ (markazi koordinatalar boshida, radiusi $|a|$ bo‘lgan yuqori yarim tekislikda joylashgan yarim aylana) va $y=1-x$ $(x\le 1)$ (boshi (1;0) nuqtada (0,1) nuqtadan o‘tuvchi nur) funksiyalar grafiklari tasvirlangan.



4-chizma

Demak, $\sqrt{a^2-x^2}=1-x$

$|a| < \frac{\sqrt{2}}{2}$ da ildiz yo'q.

$|a| = \frac{\sqrt{2}}{2}$ va $|a| > 1$ da 1 ta ildizga ega.

$\frac{\sqrt{2}}{2} < |a| \le 1$ da 2 ta ildizga ega.

Demak, tgz:

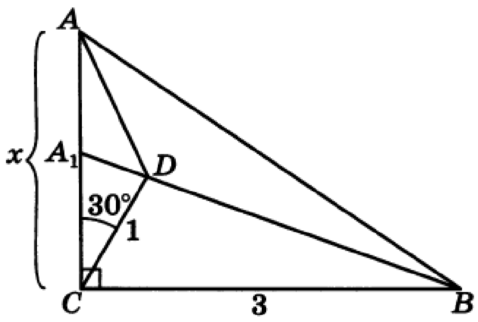
$|a| = \frac{\sqrt{2}}{2}$ da bitta qiymat qabul qiladi. ($-\frac{\sqrt{2}}{2}$ da bitta son bo'lsa, $\frac{\sqrt{2}}{2}$ da boshqa sondir).

$|a| > 1$ da bitta qiymat qabul qiladi.

$\frac{\sqrt{2}}{2} < |a| \le 1$ da 2 ta har xil qiymat qabul qiladi.

5-misol. $f(x) = \sqrt{x^2+9} - \sqrt{x^2 - x\sqrt{3}+1}$ funksiyaning eng katta qiymatini toping.

Yechish. ABC uchburchakni qaraymiz. Bunda $\angle ACB = 90^0$, $\angle ACD = 30^0$, $AC = x$, $BC = 3$, $CD = 1$ va D nuqta ABC uchburchak ichida yotadi. (5-chizma)



5-chizma

ABC uchburchakdan Pifagor teoremasiga ko'ra $AB = \sqrt{x^2+9}$.

ΔACD dan kosinuslar teoremasiga ko'ra $AD = \sqrt{x^2 - x\sqrt{3}+1}$.

$\max f(x) = \max(AB - AD) = A_1B - A_1D = DB$,

bunda $A_1 \in AC$ (agar $D \in AB$).

ΔBCD dan kosinuslar teoremasiga ko'ra

$DB = \sqrt{1^2 + 3^2 - 2\cdot 1\cdot 3\cos 60^0} = \sqrt{7}$.

Javob: $\sqrt{7}$.

6-misol. $f(x) = x\sqrt{2} + \sqrt{2x^2+2x+1} + \sqrt{2x^2-14x+25} + \sqrt{2x^2-26x+89}$ funksiyaning eng kichik qiymatini toping.

Yechish. $\vec{a}(x;x), \vec{b}(4-x;3-x), \vec{c}(x+1;x), \vec{d}(5-x;8-x)$ vektorlar va ularning modullarini qaraymiz.

$|\vec{a}| = x\sqrt{2}$.

$|\vec{b}| = \sqrt{2x^2 - 14x + 25}$.

$|\vec{c}| = \sqrt{2x^2 + 2x + 1}$.

$|\vec{d}| = \sqrt{2x^2 - 26x + 89}$.

$f(x) = |\vec{a}| + |\vec{b}| + |\vec{c}| + |\vec{d}|$.

$|\vec{a}| + |\vec{b}| + |\vec{c}| + |\vec{d}| \geq |\vec{a} + \vec{b} + \vec{c} + \vec{d}|$ bo'lgani uchun

$\min f(x) = |\vec{a} + \vec{b} + \vec{c} + \vec{d}|$.

$\vec{a} + \vec{b} + \vec{c} + \vec{d} = \overrightarrow{(10;11)}$

$|\vec{a} + \vec{b} + \vec{c} + \vec{d}| = \sqrt{221}$.

Demak, $\min f(x) = \sqrt{221}$.

Javob: $\sqrt{221}$.

7-misol. $f(x) = \sqrt{2x^2 - 4x + 2} + \sqrt{2x^2} + \sqrt{2x^2 - 2x + 2}$ funksiyaning eng kichik qiymatini toping.

Yechish. 4 ta O(0;0), A(1;1), $B\left(\frac{1-\sqrt{3}}{2}; \frac{1+\sqrt{3}}{2}\right)$, $M(x;x)$ va OM, AM va BM masofalarni qaraymiz. $OM = \sqrt{2x^2}$, $AM = \sqrt{2x^2 - 4x + 2}$, $BM = \sqrt{2x^2 - 2x + 2}$.

$f(x) = OM + AM + BM$.

ABO uchburchakning tomonlarini hisoblash qiyin emas.

$AO = BO = AB = \sqrt{2}$. Demak, ΔABO teng tomonli

$\min(AM + OM + BM) = 3R$,

R-teng tomonli uchburchakka tashqi chizilgan aylana radiusi.

$\sqrt{2} = R\sqrt{3}$ bo'lgani uchun $\min f(x) = \sqrt{6}$.

Javob: $\sqrt{6}$.

8-misol. $f(x;y;z) = \sqrt{x^2 + 1} + \sqrt{y^2 + 4} + \sqrt{z^2 + 9}$ va $x + y + z = 8$ bo'lsa, $f(x;y;z)$ funksiyaning kichik qiymatini toping.

Yechish.

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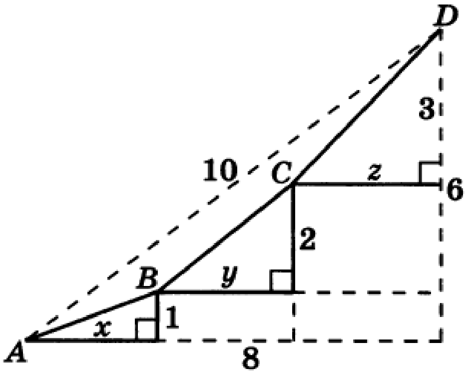

6-chizma

6-chizmaga koʻra $ABCD$ egri chiziq uzunligi 10 dan kichik emas, u holda $x+y+z-8$ da $f(x;y;z)=10$.

Javob: 10.

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